

# Application of the Vernier method with the phase shift time of flight technique for optical metrology

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## ABSTRACT

The phase shift (PS) continuous wave time of flight (PS CW TOF) method is an optical measuring technique that can be used to obtain high accuracy absolute and relative distance measurements. Optical systems based on this method can be applied as a simple and low-cost metrology solution for application in an industrial and scientific context.

A PS CW TOF range finder determines a source-target distance by comparing the phase difference of a continuously modulated signal in its source and when it is detected. Due to the cyclical properties of the phase angle, the position information is contained within an ambiguity interval. There are several established techniques that can be used to remove the ambiguity in this method, they usually rely in increasing the unambiguous range, using auxiliary measuring techniques or multiple modulation frequencies.

In this work, we present a Vernier approach applied to the dual-frequency PS CW TOF method to remove the ambiguity in a phase measurement. We validate the proposed methodology by constructing an experimental setup based on heterodyne down conversion to perform distance measurements in a [0, 5] m range. This technique allowed reaching an expanded uncertainty in the distance measurement of  $\approx 530 \mu\text{m}$  with  $k = 2$  with a modulation frequency of 1.825 GHz.

## 1. Introduction

Optical measuring techniques are commonly used for absolute distance measurement (ADM) or relative distance measurements (RDM). Techniques such as TOF are simple and easy to implement and can achieve accuracies that allow its use in a wide array of applications, from industrial metrology systems to aerospace instrumentation. Comprehensive reviews of the topic can be found in [1–5].

The phase shift (PS) continuous wave TOF (PS CW TOF) method has been shown to be able to perform relative distance measurements with a  $\mu\text{m}$  accuracy for targets in a mid-range, [1, 100] m [2,6]. The disadvantage of the direct use of this method is that absolute measurements are limited within an ambiguity range, due to the cyclical properties of the phase angle. By using a GHz modulation it is possible to achieve an ADM accuracy in the hundreds of  $\mu\text{m}$ , but the unambiguous measuring range is reduced to the cm span.

For the mentioned applications, it would be appealing to have an ADM solution based on the PS CW TOF due to its simplicity to implement and the high measurement accuracies it can achieve for mid-range measurements. In this work, we propose a different approach to solve

the ambiguity for the PS CW TOF method and we show its implementation in an experimental setup to measure a mid-range target with an accuracy in the hundreds of  $\mu\text{m}$ .

## 2. Phase shift continuous wave TOF

In a PS CW TOF method, a light source is continuously modulated in amplitude with a frequency  $f$  and directed to a target. The detected signal will then have a phase shift,  $\Delta\phi' = \phi' - \phi$ , relative to the original, which will be related to the time of flight by:

$$\Delta t = \frac{\Delta\phi'}{2\pi f} \quad (1)$$

A PS has two different components:

$$\Delta\phi' = \Delta\phi + 2\pi N \quad (2)$$

The residual PS angle,  $\Delta\phi$ , is the local difference in phase between both signals, and the ambiguity integer,  $N$ , is the number of phase cycles necessary for the phase to reach the moment it was measured, at a given modulation frequency.

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By considering that the optical signal is reflected by a target (using a mirror or a corner cube), the measured source-target distance will be half of the geometrical distance travelled by the light. So, with the phase shift as given by Eq. (2), the distance is:

$$D = \frac{D_{geo}}{2} = \frac{c}{2nf} \left( \frac{\Delta\phi}{2\pi} + N \right) \quad (3)$$

Being  $n$  the air refractive index and  $c$  the speed of light. For a given modulation frequency, there will be a maximum target distance for unambiguous phase measurement, i.e.,  $\Delta\phi = 2\pi$  rad and  $N = 0$ . This is referred as the ambiguity distance, given by:

$$\Lambda = \frac{c}{2nf} \quad (4)$$

### 3. The ambiguity problem

The different approaches to solve the ambiguity in a phase shift measurement with the PS CW TOF technique usually rely on increasing the method ambiguity range or to use auxiliary measurements to remove it. One way to overcome the ambiguity problem is just by decreasing the modulation frequency so that the ambiguity range covers the system's operating range. However, due to the inverse relation between the modulation frequency and the distance, the unambiguous range must be traded off with the desired accuracy.

Other approaches consist in using multiple modulation frequencies and combining their results to obtain the source-target distance, as suggested in [7]. By using this method, a 100 m distance was measured with frequencies of 1 MHz, 10 MHz, and 100 MHz, and it obtained a 0.12 mm uncertainty [8].

Another way is by using auxiliary measuring methods like the pulse TOF. The latter can be used to obtain a coarse measurement of the absolute distance and the PS CW TOF method to obtain a finer accuracy in the measurement [9]. By using this concept, an ADM in a range of dozens of km obtained  $2 \times 10^{-6}$  relative uncertainty [10].

Finally, the dual frequency PS CW TOF method uses two different modulation frequencies slightly shifted from each other, so that there is only one common result for both phase shift measurements and thus removing the ambiguity. This is widely used for imaging cameras [11] than can achieve sub-millimetre accuracies [12]. A similar approach for laser-based systems has been used by Fujima et al. [6] and Norgia et al. [13].

In this work, we propose a technique based in the dual frequency PS CW TOF method, that allows for the determination of the ambiguity integer in a customizable operation range. This consists in configuring the dual frequency method parameters for the desired ADM range. We performed a trade-off analysis so that it is optimized for ADM in a mid-range [1, 100] m measurements with  $\mu\text{m}$  accuracies.

### 4. An application of the Vernier method for the ambiguity measurement

An ADM with the PS CW TOF method is given by Eq. (3). When solving this equation for the residual phase shift, one obtains Eq. (5) and by plotting it with a constant modulation frequency a saw-tooth profile is revealed, due to the residual phase shift amplitude being  $[0, 2\pi]$  rad as shown in Fig. 1. With this graphical profile it is possible to obtain a way to calculate the ambiguity integer by considering the difference,  $\Delta\Phi$ , in two phase shift measurements,  $\Delta\phi$  and  $\Delta\phi'$  for frequencies  $f$  and  $f' = f + \Delta f$ , respectively, Eq. (6).

$$\frac{\Delta\phi}{2\pi} = \frac{2nf}{c} D - N \quad (5)$$

$$\Delta\Phi = \Delta\phi' - \Delta\phi \quad (6)$$

With Eqs. (5) and (6), one can write:

$$\frac{\Delta\Phi}{2\pi} = \frac{2nf'}{c} D - N' - \left( \frac{2nf}{c} D - N \right) = D \frac{2n\Delta f}{c} + \Delta N \quad (7)$$

Where  $\Delta N = N - N'$  represents the difference in ambiguity integers for a given distance, due to the way the difference in phase shift and frequency have been defined,  $N' \geq N$ , for any given distance, meaning that  $\Delta N \leq 0$ .

By rewriting Eq. (7), one obtains a way to calculate the distance:

$$D = \frac{c}{2n\Delta f} \left( \frac{\Delta\Phi}{2\pi} - \Delta N \right) \quad (8)$$

Finally, by using the last equation with Eq. (3), and considering a fixed distance, we obtain the ambiguity integer:

$$N = \frac{\left( \frac{\Delta\Phi}{2\pi} - \Delta N \right)}{\Delta f} - \frac{\Delta\phi}{2\pi} \quad (9)$$

By measuring a distance with two known different modulation frequencies, one determines three of the variables to calculate  $N$  as a function of  $\Delta N$ .

#### 4.1. Understanding $\Delta N$

For a pair of arbitrary and different frequency-phase shift points, the  $\Delta N$  between them can be 0, -1, -2, -3... depending on the  $\Delta f$  spacing, as seen in Fig. 2. For frequency shifts with different magnitudes, the measured phase shifts can have the same ambiguity integer, as seen by the pairs of frequencies  $f$  and  $f_1$ , on Fig. 2, or to be in the following integer, pairs  $f$  and  $f_2$  or  $f_3$ . Essentially, the knowledge of what values  $\Delta N$  can assume depends on the magnitude of  $\Delta f$  and the target distance,  $D$ . This means that by restricting the magnitude of  $\Delta f$  and  $D$ , we can ensure a given value for  $\Delta N$ .

#### 4.2. Limiting $\Delta f$

The easiest way to limit  $\Delta f$  is by using the phase shift difference between measurements. We define the maximum frequency shift,  $\Delta f_{\max}$ , as when  $\Delta\Phi = 0$ , meaning that the pair of phase shift measurements have been displaced by  $2\pi$ , Fig. 3. In this case one is measuring  $\Delta\phi'$  in the next ambiguity integer of  $\Delta\phi$ , meaning that  $N' = N + 1$ , hence  $\Delta N = -1$ .

Conceptually, this is analogue to the Vernier method [14,15], where information is drawn by the coincidence of two different scales. Usually, this is applied to mechanical callipers to achieve finer accuracy in displacement measurements, but in our case we use it to establish the different  $\Delta N$  one might obtain in a phase shift measurement.

Based on Fig. 3 for measurements with a frequency shift  $\Delta f < \Delta f_{\max}$ , one will find a pair of measurements at frequencies  $f$  and  $f'$  where the corresponding phase shift has the same or the next ambiguity integer when compared with the reference one,  $\Delta N = 0 \vee \Delta N = -1$ .

Since  $\Delta\Phi = 0$  implies that  $\Delta N = -1$ , by using Eq. (8) one can find the relation between the maximum frequency shift and the target distance:

$$D_{\max} = \frac{c}{2n\Delta f} (0 - (-1)) = \frac{c}{2n\Delta f_{\max}} \quad (10)$$

#### 4.3. Method

To perform an ADM with this method it is still required to know when to use  $\Delta N = 0 \vee \Delta N = -1$ . For any dual frequency measurement with  $\Delta f < \Delta f_{\max}$ , it is possible to determine when  $\Delta N = 0 \vee \Delta N = -1$  by knowing if  $\Delta\Phi$ , as it is defined, is greater or smaller than zero.

For any pair of frequencies in the colored region in Fig. 4, there can only be two outcomes for  $\Delta\Phi$ :

- $\Delta\phi' > \Delta\phi \rightarrow \Delta\Phi > 0$  - Both phase shifts are in the same ambiguity integer,  $N = N'$ , hence  $\Delta N = 0$
- $\Delta\phi' \leq \Delta\phi \rightarrow \Delta\Phi \leq 0$  -  $\Delta\phi'$  is in the next ambiguity integer when compared to  $\Delta\phi$ ,  $N' = N + 1$ , hence  $\Delta N = -1$

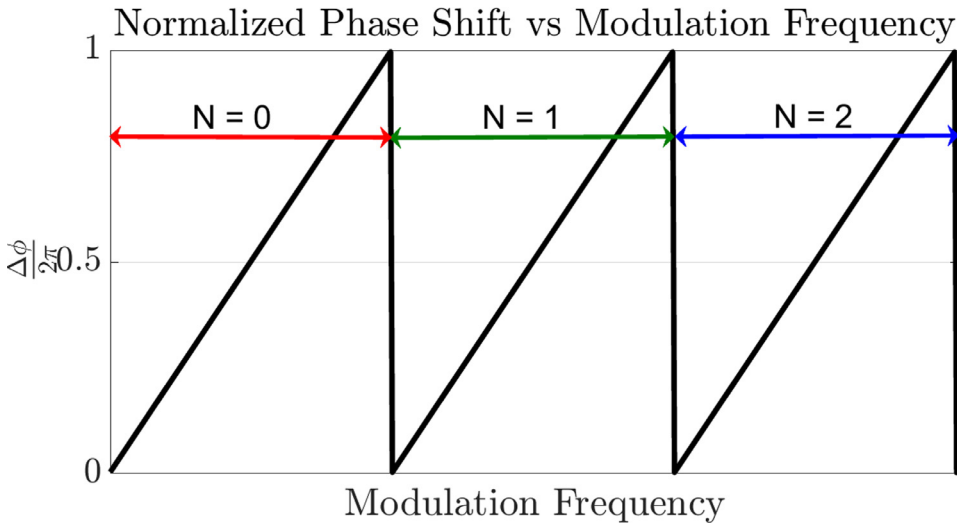


Fig. 1. Saw-tooth profile described by the residual phase shift amplitude being contained in  $[0, 2\pi]$  rad.

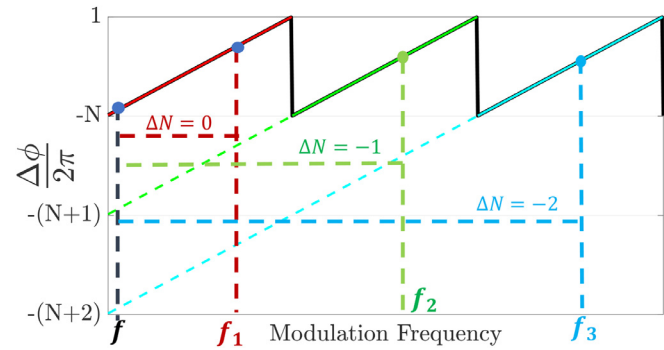


Fig. 2. Comparison of the ambiguity integers of the frequencies  $f_1$ ,  $f_2$  and  $f_3$  relative to  $f$  for a fixed distance. Showing that depending on the frequency shift relative to  $f$  different  $\Delta N$  can be found.

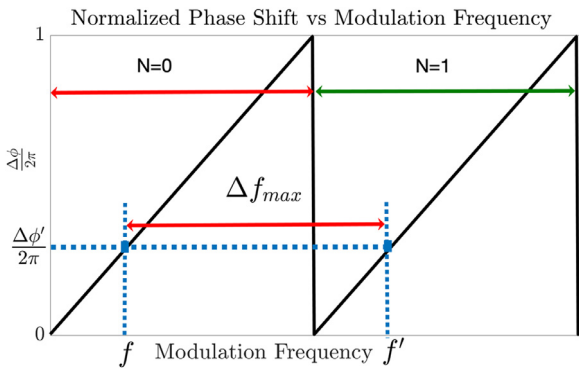


Fig. 3. For a fixed distance, when comparing two frequency-phase shift points at frequencies  $f$  and  $f' = f + \Delta f_{\max}$ , the correspondent residual phase shifts are equal to each other. This reveals that the full phase shift has been shifted by  $2\pi$ , meaning that the measurements have different ambiguity integers, hence  $\Delta N = -1$ .

The difference in phase shifts,  $\Delta\Phi$  provides a way to determine when to use  $\Delta N = 0 \vee \Delta N = -1$ .

This method is not limited by determining  $\Delta N = 0 \vee \Delta N = -1$  and measure distances smaller than  $D_{\max}$ . For other absolute positions, the same conclusions can be drawn for other  $\Delta N$ , since  $\Delta\Phi = 0$  is bound to occur cyclically as the phase shift is displaced by a multiple of  $2\pi$  rad. For example, when measuring a distance in the interval  $D_{\max} \leq$

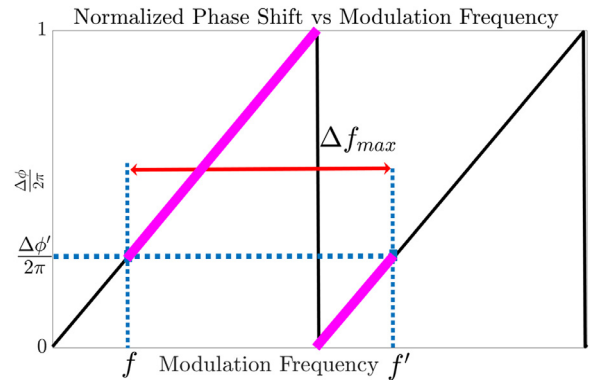


Fig. 4. For any  $\Delta f < \Delta f_{\max}$ , if  $\Delta\phi < \Delta\phi'$  it means  $N' = N$ , hence  $\Delta N = 0$ . On the other hand,  $\Delta\phi > \Delta\phi'$  then  $N' = N + 1$  and  $\Delta N = -1$ .

$D \leq 2D_{\max}$  with  $\Delta f < \Delta f_{\max}$ , one would obtain a pair of phase measurements with  $\Delta\Phi > 0 \rightarrow \Delta N = -1$  and  $\Delta\Phi \leq 0 \rightarrow \Delta N = -2$ , leading to  $\Delta N = -1 \vee \Delta N = -2$ . In this situation it is not possible to determine  $\Delta N = 0$  according to our method, meaning it is not possible to obtain information regarding a target distance in  $0 \leq D \leq D_{\max}$ . However, the unambiguous range gains an offset of  $D_{\max}$  and ADM are possible in the interval  $D_{\max} \leq D \leq 2D_{\max}$ .

To sum it up, by using the dual-frequency PS CW TOF method it is possible to measure the ambiguity integer by adjusting the maximum frequency shift  $\Delta f_{\max}$  to the corresponding measuring range. By measuring two phase shifts at frequencies  $f$  and  $f'$  our method allows for us to know when to use  $\Delta N = 0 \vee \Delta N = -1$ , solving Eq. (9) and determining the ambiguity integer. Moreover, with Eq. (3) and one of the phase shift measurements one can obtain the source to target absolute distance.

## 5. Performance analysis

This approach on the dual-frequency PS CW TOF technique allows the customization of the sensor's operating range by adjusting the unambiguous measuring range with the correspondent  $\Delta f$ , as described in the last section. In Fig. 5 is shown the relation between the maximum frequency shift and the maximum amplitude for the unambiguous range, based on Eq. (10).

This method is valid in essence for any measurement range, although in practice, the measurement uncertainty associated to  $N$  increases with the distance, this means that at a given point it will not be possible to correctly identify the ambiguity integer for that measurement. This means

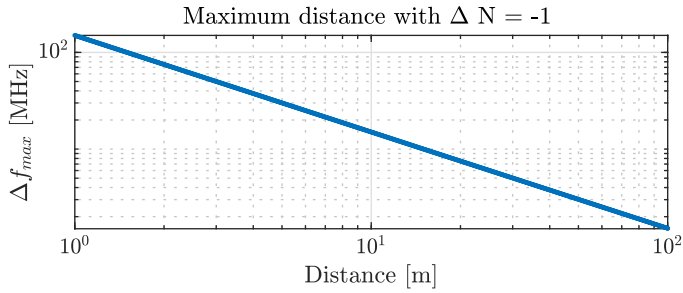


Fig. 5. Relation between the method's unambiguous measuring distance and  $\Delta f_{\max}$ .

that the method performance falls behind other well-established techniques, such as pulse TOF that can achieve ADM in a comparable range with mm accuracy, being fundamentally limited by the pulse width and the accuracy of the timing discriminator [1–3].

Hence, we establish a requirement in the expanded uncertainty of the ambiguity integer,  $U_N < 0.5$  with  $k=2$ . This limitation is critical to ensure that we only find one integer value in the validity range of an ambiguity measurement, allowing to pinpoint the exact  $N$  for that position.

We did a brief uncertainty analysis for the distance and ambiguity integer measurement with Eqs. (11) and (12). In this evaluation, we considered the uncertainty contributors: the phase shift measurement uncertainty, the frequency stability of the oscillator that is modulating the light source and the air refractive index.

In our case, the phase and frequency contributions for the uncertainty budget were established by the instruments that we had available to amplitude modulate the laser and to perform phase measurements,  $u_{\Delta\phi} = 0.87^\circ$  and  $u_f = 1.8$  kHz. The air refractive index was calculated with the NIST metrology toolbox [16] with the Ciddor Equation and an expanded uncertainty of  $U_n = 2.3 \times 10^{-8}$ ,  $k=2$ , was obtained with the parameters:  $\lambda_{vac} = 1550$  nm,  $T_{ambiente} = 20$  C°,  $P = 101.325$  kPa and a relative humidity of 50%.

$$u_D^2 = \left(\frac{\partial D}{\partial \Delta\phi}\right)^2 u_{\Delta\phi}^2 + \left(\frac{\partial D}{\partial f}\right)^2 u_f^2 + \left(\frac{\partial D}{\partial n}\right)^2 u_n^2 \quad (11)$$

$$u_N^2 = \left(\frac{\partial N}{\partial \Delta\phi}\right)^2 u_{\Delta\phi}^2 + \left(\frac{\partial N}{\partial \Delta\Phi}\right)^2 u_{\Delta\Phi}^2 + \left(\frac{\partial N}{\partial f}\right)^2 u_f^2 + \left(\frac{\partial N}{\partial \Delta f}\right)^2 u_{\Delta f}^2 \quad (12)$$

The results in Fig. 6 are for a fixed  $\Delta f = 25$  MHz and they show that the ambiguity integer uncertainty requirement,  $U_N < 0.5$ , can be achieved for a range of hundreds of meters, depending on the modula-

tion frequency. However, there is a trade-off between the range where the  $N$  uncertainty criterion is achieved and the ADM uncertainty. Also, we figured that by adjusting  $\Delta f$  it is possible to further trade-off with  $U_N$  and the sensor unambiguous range amplitude, according to Fig. 5.

With the correct trade-offs between the measuring range, the frequency shift and the modulation frequency, this method can achieve relative uncertainties in the order of  $[10^{-6}, 10^{-5}]$  for a  $[0, 100]$  m span, Fig. 7. Hence, this technique can be used for measuring ranges comparable with pulse TOF solutions, but with a relative uncertainty with an order of magnitude lower. This can be beneficial when applying this method in scientific and industrial processes that require a high accuracy solution operating within the mentioned span.

It is important to notice that it is necessary to determine a reference position for the sensor ADM. The reference position was measured with an uncertainty  $u_{D_{REF}}$  that will be approximately the same magnitude as the uncertainty in the measurement of an interest position,  $u_{D_i}$ . This increases the total uncertainty per position,  $u_{D_i}$  of an ADM with the sensor by  $\approx \sqrt{2}$ :

$$u_{D_i}^2 = u_{D_i'}^2 + u_{D_{REF}}^2 \quad (13)$$

Being  $i = 1, 2, 3, \dots$  the index for the different measured positions.

This increase in the measurement uncertainty can be easily overcome by using another ADM instrument, with an accuracy better than of the presented method (hundreds of nm), to calibrate the presented sensor reference. This will decrease substantially the reference position uncertainty and ensures that the biggest uncertainty contributor for an ADM comes from the measurement of the interest position.

## 6. Measurement solution

The developed methodology was tested by using an electro-optic scheme based on heterodyne down-conversion (HdC), as it was presented in [13]. HdC provided a solution to down-convert the GHz frequency of the reference and detected signals without losing any phase information. With this technique, we have all the advantages of using a GHz modulation frequency, without the need to use specific and expensive instrumentation.

The setup was designed to operate in a  $[0, 5]$  m range with an initial modulation frequency of 1.8 GHz that is shifted by  $\Delta f = 25$  MHz. According to the simulations from the last section, these parameters allowed for a  $U_N \approx 0.49$  and  $U_D \approx 580$   $\mu\text{m}$  (@ 1.825 GHz). We decided to use a  $\Delta f$  smaller than  $\Delta f_{\max}$  for this range, because overall it provided a  $\text{SNR} \approx 10$ , allowing a more accurate phase measurement than when using  $\Delta f_{\max} \approx 30$  MHz.

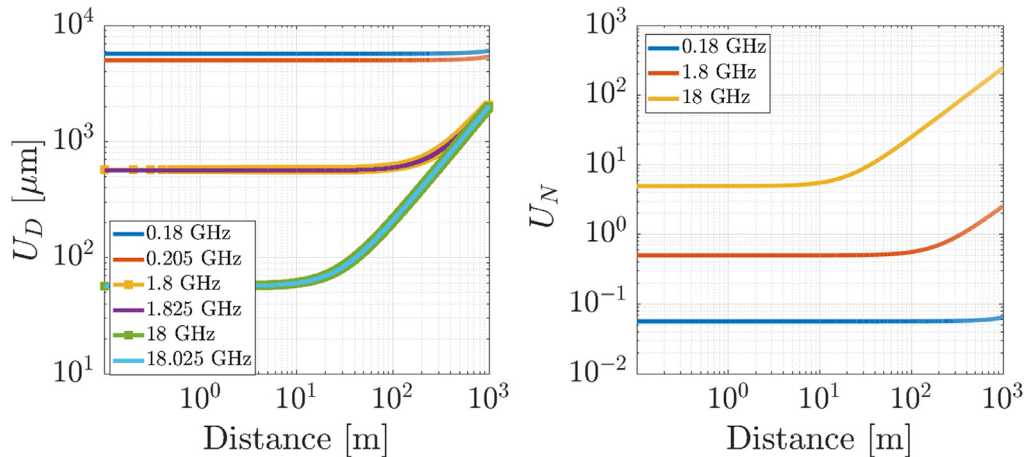
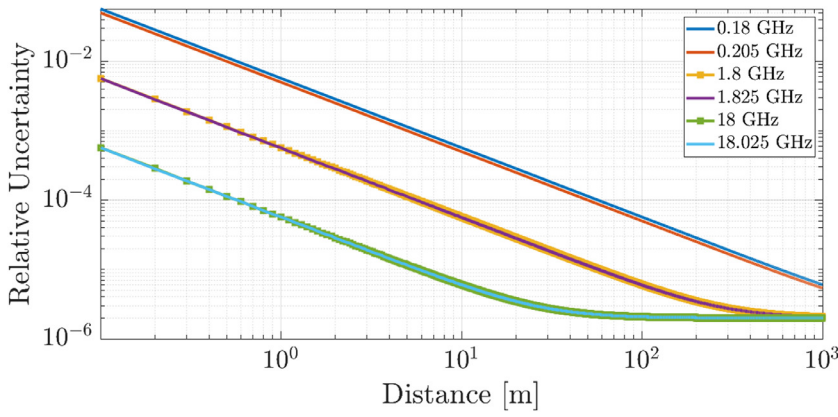
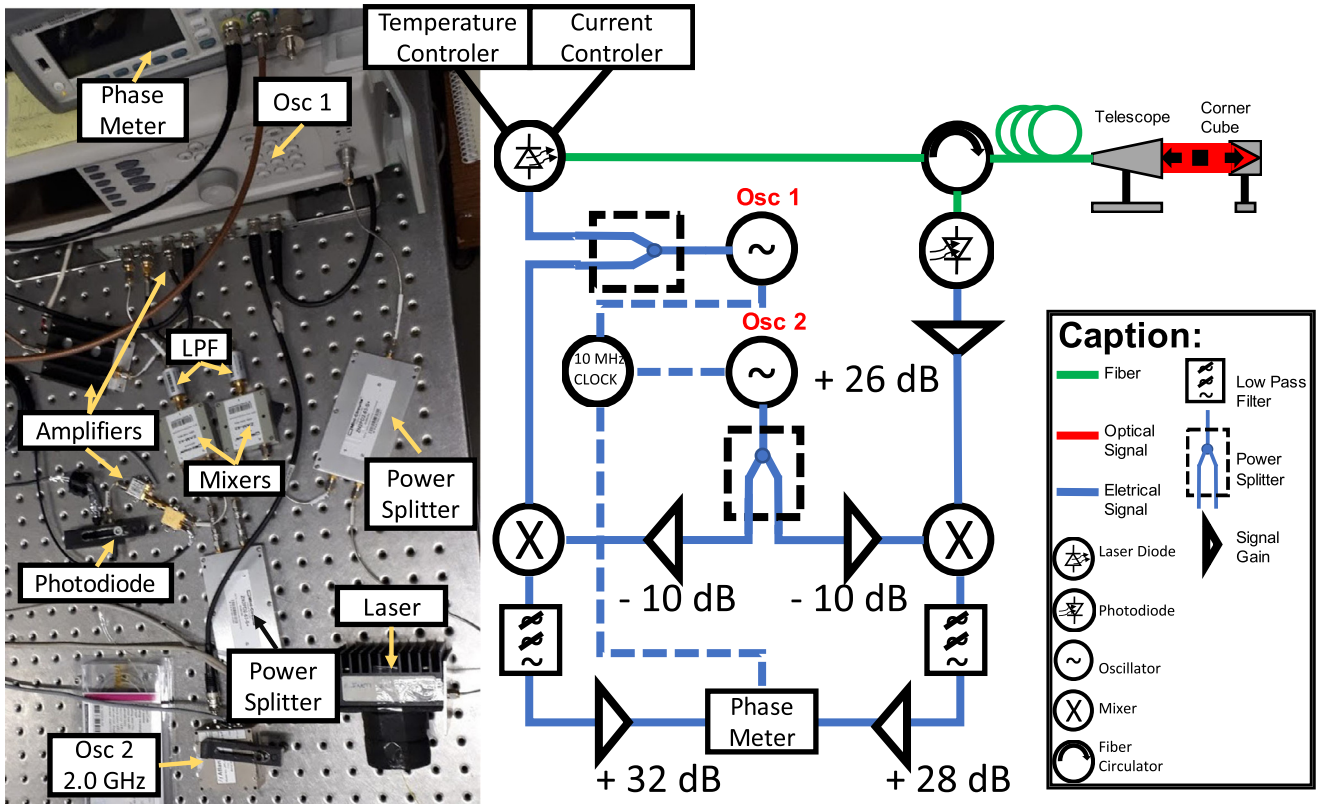


Fig. 6. Analytical simulation of the expanded uncertainty of the ambiguity integer  $N$  and ADM with  $k=2$  and 95 % confidence interval. Parameters:  $D = [0, 1000]$  m with a 0.1 m step,  $\Delta f = 25$  MHz,  $u_f = 1.8$  kHz and  $u_{\Delta\phi} = 0.87^\circ$ .





**Fig. 7.** Analytical simulation of the relative uncertainty for an ADM. Parameters:  $D = [0, 1000]$  m with a 0.1 m step,  $\Delta f = 25$  MHz,  $u_f = 1.8$  kHz and  $u_{\Delta\phi} = 0.87^\circ$ .



**Fig. 8.** Left: Photograph of the bench-top electronic part of the experimental setup. Right: Full schematic of the experimental setup.

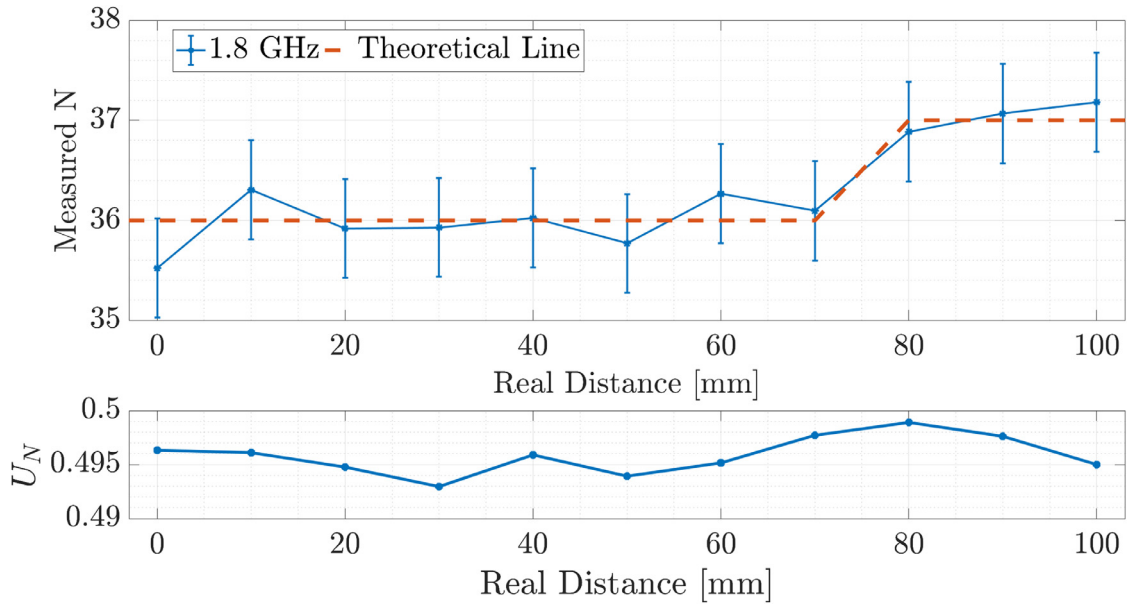
We implemented a experimental setup shown in Fig. 8 that consists of two oscillators, *Osc 1* and *Osc 2* that are involved in two different HdC processes. *Osc 1* is a tunable RF signal generator (Anritsu MG3690C) and it is responsible for amplitude modulating the laser diode. *Osc 2* is a 2.0 GHz phase lock oscillator with an internal reference clock (Atlantic Microwave) that is used as the local oscillator, to down-convert to the MHz range the laser modulation signal and the one detected by the photodiode. By measuring the phase difference between the resulting signals from the mixing process the source-target distance is obtained.

We performed short and mid-range measurements, [0,100] mm and [4.808,4.818] m, by displacing a corner cube on top of a Thorlabs NRT Series translation stage. This has a [0, 100] mm range and 1  $\mu$ m of bi-directional repeatability [17], with such high accuracy compared with the expected one for a 1.8 GHz with our method, we can consider the stage displacements as exacts.

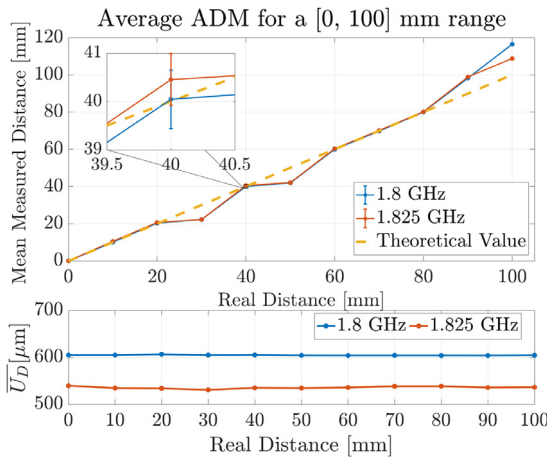
## 7. Experimental results and discussion

To evaluate experimentally the general assumptions for the measurement solution presented in the previous sections, we started by performing measurements in an [0,100] mm range. By sweeping this interval with a 10 mm step and measuring the ambiguity integer for each position, we would have to detect a change in  $N$  since its ambiguity range is  $\approx 80$  mm. If one can do so, then our proposed method to remove the ambiguity is validated.

We displaced the corner cube and measured a pair of phase shifts for each position and obtained  $N$ . The results in Fig. 9 show that the resulting interval for the ambiguity integer measurement at each position contains only one integer value. This allows for the exact determination of this quantity, leading to the detection of a change in  $N$  along with the displaced positions.



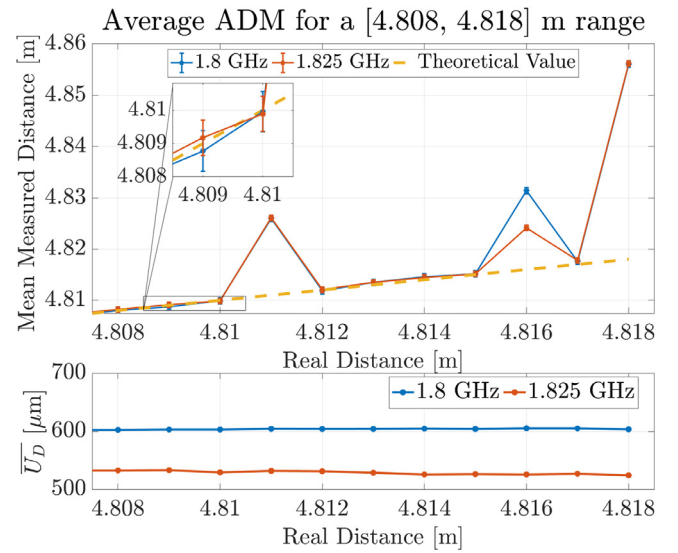
**Fig. 9.** Experimental results for the ambiguity integer measurement for an ADM in a range of [0, 100] mm with a 10 mm step. Additionally, it is presented the expanded uncertainty with a confidence interval of 95 % and a coverage factor of  $k = 2$ .



**Fig. 10.** Average obtained ADM for each position in a [0, 100] mm range and its average expanded uncertainty with a coverage factor of  $k = 2$ , meaning a confidence interval of 95 %.

Afterwards, we repeated the procedure but this time we did eleven ADM measurements per position and referenced each to the measurement at the 0 mm mark of the translation stage. The increased sample size of the phase shift revealed sets of measurements where we did not determine the correct ambiguity integer for that position, which lead to an ADM with an offset of one ambiguity range ( $\approx 80$  mm). Further investigation of this issue led us to realize that the wrong phase shift measurements had an offset compared to the ones from the same set. We cannot fully describe the origin of this random noise effect, but it probably has to do with some electrical or optical contributions from the setup. Further work is required to determine exactly the source of this error and find ways to mitigate it.

The results in Fig. 10 for the 1.825 GHz frequency have an ADM expanded uncertainty of  $U_D \approx 530 \mu\text{m}$  ( $k=2$ ). As one can notice, the magnitude of the measured  $U_D$  is different from the one obtained in the analytical simulations. This was due to the contribution of the phase shift uncertainty being overestimated in the theoretical calculation, with



**Fig. 11.** Average obtained ADM for each position in a [4.808, 4.818] m range and its average expanded uncertainty with a coverage factor of  $k = 2$ , meaning a confidence interval of 95 %.

a value of  $u_{\Delta\phi} = 0.87^\circ$ . In the experimental application, we verified that at 1.825 GHz we obtained a  $u_{\Delta\phi} = 0.82^\circ$  which led to a  $U_D \approx 530 \mu\text{m}$ .

The sensor was also tested with the same procedure in a  $\approx 4.8$  m range by doing [0, 10] mm displacements with a 1 mm step. This set of tests serves to understand the sensor response to an ADM at a greater distance than the previous measurements. Due to space limitations in our laboratory, we extended the OPL by using optical fibre instead of doing it in free-space. The fibre had a refraction index of  $n_{\text{fibre}} = 1.4682$  and a length of  $\approx 3.2$  m, leading to an added path of  $\approx 4.8$  m. Since the fibre length was measured with a ruler it is a rough approximation of its actual length. Therefore, this OPL addition only serves to add an offset to the [0, 10] mm interval that was swept.

The results, shown in Fig. 11, reveal that the expanded uncertainty in the ADM measurement at the mid-range is in the same magnitude as the short-range ( $\approx 530 \mu\text{m}$ ). This takes place since we expanded the

OPL with an optical fibre and so the majority of the signal was efficiently propagated through the medium. The only exception was a path of  $\approx 40$  cm, where the optical beam was expanded in the telescope and reflected on the corner cube.

When accounting for measurements in free-space in air, the heterogeneity of the medium refractive index is a big contributor to the deterioration of the SNR of the detected signal, leading to greater measurement uncertainties. Nevertheless, this sensor can ensure accuracies in the hundreds of  $\mu\text{m}$  for applications where the medium refractive index does not impose a constraint, such as in space-based systems.

## 8. Conclusion

In this work, we identified the PS CW TOF technique as an appealing technique for high accuracy distance measurements, due to its simplicity to implement with off-the-shelf components and the capability of reaching hundreds of  $\mu\text{m}$  for [1, 100] m operating intervals. An ADM sensor with these characteristics is attractive as a solution for metrology systems in scientific and engineering contexts.

We developed a Vernier method with the established dual-frequency PS CW TOF technique to obtain a new approach to remove the ambiguity in a particular measuring range. We showed that the uncertainty requirement of  $U_N < 0.5$  is what fundamentally limits the maximum measuring range of the method. There is a trade-off between the measuring range, the frequency shift magnitude and the modulation frequency so that high accuracy measurements can be achieved. This method reveals to be a promising solution for mid-range measurements [1, 100] m to obtain relative uncertainties of  $[10^{-6}, 10^{-5}]$ .

Overall, the constructed experimental setup was able to validate our measurement concept. The achieved uncertainties of  $U_D \approx 530 \mu\text{m}$  with  $k=2$  (@ 1.825 GHz) are acceptable for a low-cost experimental setup with off-the-shelf components and benchtop instrumentation. Since this design is based on the electrical HdC of the signal, it opens an opportunity to be miniaturized and integrated into a single board range finder. This integrated solution can reduce RF interference noise between components and it also offers a compact solution for applications where volume and weight are critical, such as space and aerospace.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## CRediT authorship contribution statement

**Nuno M. Gonçalves:** Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Writing – original draft, Writing – review & editing, Visualization. **Manuel Abreu:** Conceptualization, Methodology, Investigation, Resources, Writing – original draft, Supervision, Project administration, Funding acquisition. **D. Castro Alves:** Conceptualization, Methodology, Resources, Writing – original draft.

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